Analysis of Smart Composite Material Structures

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***Abstract*— Composite materials have wide range of applications in the aerospace and defence structures. A finite element formulation is developed to model the response of buckling of composite plates with piezoelectric sensor and actuator layers. The stress equations of motion are derived using the variational principle with respect to the total structural and electrical potential energy. Semiloof shell element was used to study the behaviour of the composite plates subjected to mechanical loading. The material properties: Isotropic, Piezoelectric and composite were considered. The analytical solution is validated by comparing the available results in the literature. New results are presented for the effect of piezoelectric material on the buckling for the cross ply laminated plates.**

***Keywords*— Composite Material; Semiloof; Piezoelectric; Finite Element, Plates and Shells, Cross-ply**

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I. INTRODUCTION

An increase in the need for lightweight structures requires the development of new kinds of materials. Composite materials have been observed to improve various attributes such as strength, stiffness, fatigue, corrosion resistance, wear resistance, weight, acoustical insulation etc., and are extensively used for several applications, like aerospace structures, defence structures, civil, mechanical and other industries that require high specific mechanical properties. Some composite materials such as fibre reinforced laminates, generally have high strength and stiffness. The availability of such functional materials and the feasibility of bonding them to the composite structures has instigated the use of smart structural concepts for high-performance structural applications. Moreover, with such features incorporated in the structures, it is feasible to achieve various technological advances. Piezoelectric materials such as PVDF (Polyvinylidene Fluoride) films are bonded to the laminate layers and are employed as actuators and sensors. These smart materials have found a way in many structural applications such as vibration control, buckling control, noise control and structural health monitoring.

Benjeddou [1] surveyed and discussed the advances and trends in the formulations and applications of the finite element modelling of adaptive structural elements. Modeling of smart piezoelectric laminate shell structures with finite elements was carried out by Piefort and Preumont [2]. A general finite element formulation using the layer-wise theory for the analysis of laminated plate structures with piezoelectric layers or patches was explored by Semedo Garção et *al.*, [3]. Andrew [4] has used piezo-ceramic actuators for the active buckling control, thereby analysing the strength and stability of many structures. Hesham *et al.*, [5] performed a numerical study to highlight the effect of bonded prestressed composite patches on the fracture parameters, such as crack tip opening displacement and the plastic zone. In the same way, mathematical analysis and numerical stimulation of smart composite structures with the concept for laminated thin finite shell elements with active and passive layers considering some three approaches were introduced by Gabbert *et al.*, [6]. A finite element development for generally shaped piezoelectric active laminates using linear approach was studied by Marinković *et al.*, [7], consequently, functionally graded and laminated piezoelectric cantilever actuators subjected to a constant voltage was researched and analyzed by Huang *et al.*, [8]. Finite Element Analysis of piezoelectric structures with nonlinear material behaviour was investigated by Lammering and Mesecke [9], where Non-linear constitutive equations for piezoelectric material were presented considering a linear dependence of the piezoelectric constants on the mechanical strains. A Reissner-Mindlin type modelization of piezoelectric plates was considered in a suitable variational framework by Auricchio *et al.*, [10]. A finite element scheme that could approximate the solution was then proposed and theoretically analysed. A finite element modelling using Piezoelectric active structures was discussed by Piefort [11], in which the finite element approach to the smart structures and their applications were studied. An accurate modelling of the electric field was performed by Marinković *et al.*, [12], to demonstrate that a quadratic distribution of the electric field is adequate for the piezoelectric patch that exhibits kinematics described by the First-order two-dimensional theory. Buckling analysis of Piezoelectric composite plates using NURBS-based isogeometric finite elements and higher-order shear deformation theory was investigated and put forward by

Phung-Van *et al.*, [13]. Hessamddin [14] conducted a buckling analysis of a three-layered rectangular plate embedded with piezoelectric layers. An analytical method to analyze the buckling of piezoelectric coupled plates with different boundary conditions based on the first order shear deformation plate theory was employed in the research work. The effect of modified couple stress theory on buckling and vibration analysis of functionally graded double-layer boron nitride piezoelectric plate based on the classical plate theory was researched by Mohammadimehr and Mohandes [15]. Geng [16] investigated the enhancement of the dynamic buckling load and analysis of active constrained layer damping with extension and shear mode piezoceramic actuators, studying numerically, by the finite element method, transient three-dimensional electro-elastic deformations of a graphite-epoxy square plate sandwiched between two piezoceramic (PZT) layers considering the geometric and material nonlinearities. Vel and Batra [17] conducted an analysis on bimorph elastic plates with distributed or segmented piezoelectric layers and accordingly analysed using the classical laminated plate theory, the first-order shear deformation theory.

The theoretical and experimental study of piezoelectric materials has been the focus of considerable research for many years now. Researches on thin plates and shells are less when compared to those on degenerated shells. Therefore, in engineering applications, to overcome the problems of multiple joints and sharp corners, an element which uses the isoparametric shell theory in addition to the Discrete Kirchhoff's hypothesis was developed by Irons [18], known as the Semiloof shell element.

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II. FINITE ELEMENT FORMULATION

The constitutive equation for the piezoelectric actuator material is given as [19],

$$\sigma = [\bar{Q}]\,(\bar{e} - \Lambda) \qquad (1)$$

where $[\bar{Q}]$ is Transformed reduced stiffness matrix, Λ is the actuation strain vector $\Lambda = d_{31}\,\frac{V}{Tp}$, V is the voltage, T_p is the piezoelectric thickness. d_{31} is the piezoelectric constant.

The global constitutive equations of the laminate that relate the resultant in-plane forces {N} and the bending moment {M} to the mid-plane strain {ϵ} and curvature {κ} and the potential applied to the various electrodes after integrating is obtained as,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} - \begin{Bmatrix} N_\Lambda \\ M_\Lambda \end{Bmatrix} \qquad (2)$$

$$\{F_\Lambda\} = \begin{Bmatrix} N_\Lambda \\ M_\Lambda \end{Bmatrix} \qquad (3)$$

The first term in the right-hand side of the Eqn. (2) is the classical stiffness matrix of a composite laminate, where the extensional stiffness matrix [A], the bending stiffness matrix [D] and the extension/bending coupling matrix [B] are related to the individual layers per the classical relationships:

$$([A], [B], [D]) = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{Q}]\ (1, Z, Z^2)\ dz \qquad (4)$$

$$[A] = \sum_k [\bar{Q}]_k\,(z_k - z_{k-1})$$
$$[B] = \frac{1}{2}\sum_k [\bar{Q}]_k\,(z_k^2 - z_{k-1}^2)$$
$$[D] = \frac{1}{3}\sum_k [\bar{Q}]_k\,(z_k^3 - z_{k-1}^3) \qquad (5)$$

The second term on the right-hand side of Eqn. (2) expresses the piezoelectric loading.

The induced force, N_Λ and moment, M_Λ is expressed as,

$$N_\Lambda = \sum_k [\bar{Q}]\Lambda_k\,(h_{k+1} - h_k)$$
$$M_\Lambda = \sum_k [\bar{Q}]\Lambda_k\, h_{k+1}^2 - h_k^2) \qquad (6)$$

The minimum total potential energy gives rise to the governing equations for the equilibrium states [20],

$$\{[K] + [K_G]\}\ \{q\} = \{F\} + \{F_\Lambda\} \qquad (7)$$

To establish the critical buckling state corresponding to neutral equilibrium condition, the second variation of the total potential energy must be equal to zero.

Hence, $|[K] + \lambda[K_G]| = \{0\}$ (8)

Applying Hamilton's principle yields the equation of motion for the structure under free vibration,

$$\{[K] - \omega^2[M]\}\ \{q\} = \{0\} \qquad (9)$$

III. RESULTS AND DISCUSSION

Based on the Semiloof shell element formulation, a computer program "COMSAP" was developed by Thangaratnam *et al.*, [21] Piezoelectric property is added to the program and is extended to Isotropic, Composite and Functionally Graded Materials (FGM). The accuracy of the program is verified by using relevant results available in the literature.

A laminated Glass-Epoxy cantilevered plate consisting of PVDF piezoelectric polymers on the top and bottom layers is analysed [22]. The material property of polyvinylidene Fluoride is listed in Table I. The material properties of the base plate are listed in Table II. The plate is analysed with clamped boundary condition. Results are obtained for antisymmetric and symmetric cross-ply laminates varying with 2, 4, 6 & 8 laminated layers. The buckling loads are calculated for varying voltage between 0 – 700 Volts and voltage vs buckling load graph is plotted for the 2, 4, 6, 8-layer cross-ply symmetric and antisymmetric laminates, which is as given in Fig. 1 and Fig. 2. Similarly, the same is plotted for the 1,3,5,7-layer cross-ply symmetric laminate Fig. 3.

TABLE I
MATERIAL PROPERTY FOR PVDF

Property	**PVDF**
Elastic Modulus, E (10^9 Pa)	2.0
Poisson's Ratio, υ	0.29
Shear Modulus, G (10^9 Pa)	0.8
Piezoelectric Charge Constants (10^{-12} m/V)	

d_{31}	23.0
d_{32}	3.0

TABLE III
MATERIAL PROPERTY FOR GLASS-EPOXY

Property	Laminate
Young's Modulus in direction 1, E_1	54 Gpa
Young's Modulus in direction 2, E_2	18 Gpa
Shear Modulus, G	9 Gpa
Poisson's ratio, υ	0.25

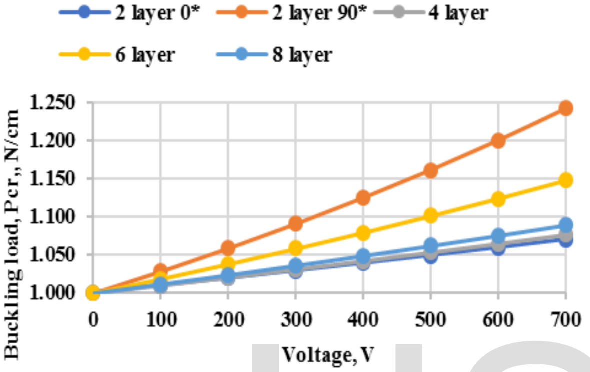

Fig. 1 Voltage vs Buckling Load for Cross-Ply Symmetric Laminates for 2,4,6,8 layers

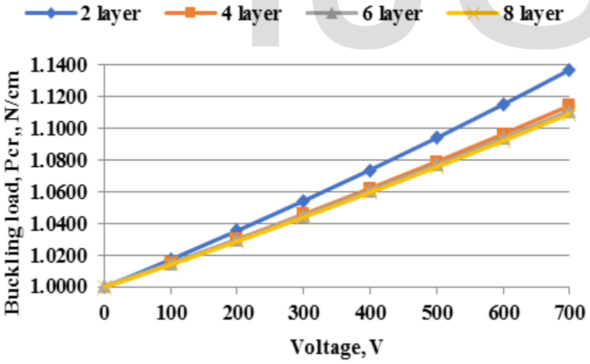

Fig. 2 Voltage vs Buckling Load for Cross-Ply Antisymmetric Laminates for 2,4,6,8 layers

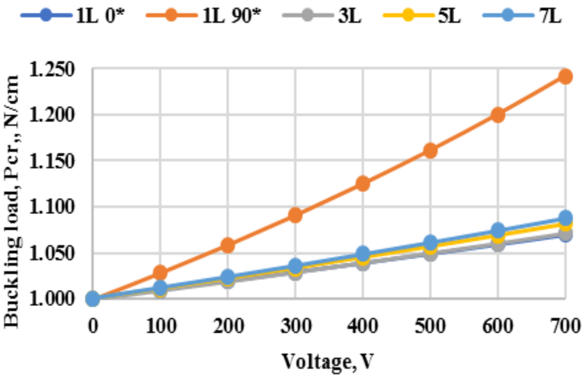

Fig. 3 Voltage vs Buckling Load for Cross-Ply Symmetric Laminates for 1,3,5,7 layers

IV. CONCLUSIONS

The formulation of governing equation for mechanical stress, free vibration, mechanical buckling analysis is based on finite element method, utilizing Semiloof shell element. From the results obtained in this work, it is evident that using piezoelectric properties along with Semiloof elements with the assistance of COMSAP package proves to be efficacious. Nevertheless, the obtained results are also found to be in par with the existing literature.

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